Published by Institute of Physics Publishing for SISSA

Received: *July 2, 2006*
Accepted: *July 31, 2006*
Published: *August 29, 2006*

Flavor changing effects in family nonuniversal Z' models

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Abstract: Flavor-changing and CP-violating interactions of Z' to fermions are generally present in models with extra U(1) gauge symmetry that are string-inspired or related to broken gauged family symmetry. We study the consequences of such couplings in fermion electric dipole moments, muon $g-2$, and K and B meson mixings. From experimental limits or measured values, we constrain the off-diagonal Z' couplings to fermions. Some of these constraints are comparable or stronger than the existing constraints obtained from other observables.

Keywords: Beyond Standard Model, CP violation, B-Physics.

Contents

1. Introduction

Additional heavy neutral Z' gauge bosons have been extensively studied in the literature. They arise naturally in grand unified models, superstring-inspired models [1] and in models with large extra dimensions [2]. There are stringent limits on the mass of an extra Z' from collider search experiments at the Tevatron [3]. Precision data put limits on the Z-Z' mixing angle θ [4]. Although these limits are model-dependent, the typical constraints are $M_{Z'} > O(500\,\text{GeV})$ and $\theta < O(10^{-3})$. Most studies have assumed flavor universal Z' couplings. However, in intersecting D-brane constructions, it is possible to have family nonuniversal Z' couplings. In extensions of the Standard Model (SM) with gauged family symmetry, nonuniversal Z' couplings also arise naturally [5]. These couplings generally lead to flavor-changing and CP-violating Z' vertices, when quark and lepton mixing is taken into account. Flavor violating Z couplings can be induced through Z-Z' mixing. Experimental observables in the flavor changing and CP violating processes may be used to put constraints on Z' couplings. One of the important searches that the Large Hadron Collider (LHC) will undertake is to look for Z' bosons. It is therefore crucial to explore the parameter space of the allowed couplings of such Z' bosons.

In a recent paper [6], Langacker and Plümacher have investigated the consequences of family nonuniversal Z' gauge boson. They have considered several processes including $Z \to \bar{q}_i q_j$, $l_i \to l_j l_k \bar{l}_m$, μ-e conversion, radiative decays of $\mu \to e\gamma$ and $b \to s\gamma$, and meson decays, etc. In this paper we extend their analysis to include constraints from electric dipole moments (EDMs) of electron and neutron and muon $g-2$. We re-analyze mass difference and CP violation in K-$\bar{K}$ mixing to emphasize the enhanced contributions from left-right mixing terms. For B_d mixing, we find that it is important to include an independent observable, that is not affected by the Z' effects, to improved the constraints.

In the Standard Model, the fermion EDMs are generated at three-loop or higher orders. The predicted value of less than $10^{-33}\mathrm{e}\cdot\mathrm{cm}$ is several orders of magnitude lower than the most stringent bounds coming from electron and neutron EDM measurements. However, in extensions of the SM, such as Z' models with family nonuniversal couplings, additional weak phases will allow fermion EDMs to be generated at the one-loop level, and thus they can be dominant. Therefore, we can use EDM measurements to constrain Z' flavor-changing couplings.

By exchanging a Z' boson, oscillations of K, B_d and B_s mesons can occur via tree-level diagrams, as compared to one-loop box diagrams in the SM. Some of the operators that occur in the K-$\bar{K}$ mixing are enhanced so that very strong limits can be obtained from ΔM_K and ϵ_K measurements. In the B_d system, the limit on $|V_{ub}|$, the measurements of ΔM_{B_d} and $\sin 2\beta$ and the recent limit on $|V_{td}/V_{ts}|$ obtained from $b \to d\gamma$ and $b \to s\gamma$ [7] can be combined to provide strong constraints on the Z' flavor-changing couplings. The recent measurements of ΔM_{B_s} at both DO [8] and CDF [9] have generated much interest in flavor mixing in B mesons [10]. Several papers have studied B_s mixing in the context of Z' models [11 − 14]. Although the ratio $|\Delta M_{B_d}/\Delta M_{B_s}|$ provides the best determination of $|V_{td}/V_{ts}|$ in the SM, when new physics effects enter both mass differences, the ratio does not have an advantage over the individual mass difference in constraining new physics variables. We will discuss this point in more detail.

The paper is organized as the following: after the introduction, we briefly describe our notations in section 2. We discuss fermion EDMs in section 3, K-$\bar{K}$ mixing in section 4, and B-$\bar{B}$ mixing in section 5. We conclude in section 6.

2. Formalism

We follow closely the formalism in ref. [6]. In the gauge eigenstate basis, the neutral current Lagrangian can be written as

$$\mathcal{L}_{\mathrm{NC}} = -eJ^{\mu}_{\mathrm{em}}A_{\mu} - g_1 J^{(1)\,\mu} Z^0_{1,\mu} - g_2 J^{(2)\,\mu} Z^0_{2,\mu}\,, \tag{2.1}$$

where Z^0_1 is the $\mathrm{SU}(2)\times\mathrm{U}(1)$ neutral gauge boson, Z^0_2 the new gauge boson associated with an additional Abelian gauge symmetry and g_1 and g_2 are the corresponding gauge couplings. The current associated with the additional $\mathrm{U}(1)'$ gauge symmetry can be generally written as

$$J^{(2)}_{\mu} = \sum_{i,j} \overline{\psi}_i \gamma_\mu \left[\epsilon^{(2)}_{\psi_{L_{ij}}} P_L + \epsilon^{(2)}_{\psi_{R_{ij}}} P_R\right] \psi_j\,, \tag{2.2}$$

where $\epsilon^{(2)}_{\psi_{L,R_{ij}}}$ is the chiral coupling of Z^0_2 with fermions, with i and j run over quark flavors, and similarly for leptons. Flavor changing effects arise if $\epsilon^{(2)}$ are nondiagonal matrices. If the Z^0_2 couplings are diagonal but family-nonuniversal, flavor changing couplings are induced by fermion mixings. When fermion Yukawa matrices h_ψ are diagonalized by unitary matrices $V^{\psi}_{R,L}$

$$h_{\psi,\mathrm{diag}} = V^{\psi}_R\, h_\psi\, V^{\psi\,\dagger}_L\,, \tag{2.3}$$

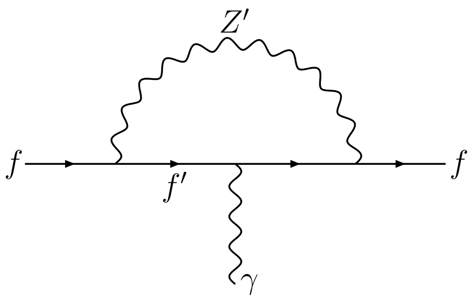

Figure 1: Feynman diagram of fermion EDMs at one-loop level through flavor changing complex $Z'ff'$ coupling.

the current associated with Z_2^0 is rewritten in the fermion mass eigenstate basis

$$J_\mu^{(2)} = \sum_{i,j} \overline{\chi}_i \gamma_\mu \left[B_{ij}^{\psi_L} P_L + B_{ij}^{\psi_R} P_R \right] \chi_j \,, \tag{2.4}$$

with

$$B_{ij}^{\psi_L} \equiv \left(V_L^\psi \epsilon_{\psi_L}^{(2)} V_L^{\psi\dagger} \right)_{ij} \,, \qquad \text{and} \qquad B_{ij}^{\psi_R} \equiv \left(V_R^\psi \epsilon_{\psi_R}^{(2)} V_R^{\psi\dagger} \right)_{ij} \,. \tag{2.5}$$

In the following sections we simplify $B_{ij}^{\psi_{L,R}}$ as $B_{ff'}^{L,R}$, with f and f' specifying the flavors of quarks and leptons explicitly and the L, R superscripts indicating left-handed or right-handed couplings. For example, $B_{13}^{d_L}$ will be written as B_{db}^L. In general, $B_{ff'}^L$ and $B_{ff'}^R$ can be complex and have independent phases.

3. Electric dipole moments

In the SM, for a dipole operator, the weak phase exactly cancels out in the one-loop diagrams. Hence there is no contribution to fermion EDMs at the one-loop level. In comparison, there can be independent phases involving the left-handed and the right-handed Z' couplings. These complex phases can contribute to fermion EDMs through the one-loop diagram shown in figure 1, where f and f' indicate fermions of different flavors.

For fermion f, the contribution to its EDM from figure 1 is evaluated to be,

$$d_f = -\frac{1}{16\pi^2} g_2^2 Q_f e \frac{m_{f'}}{m_{Z'}^2} \mathrm{Im}(B_{ff'}^{L}{}^* B_{ff'}^R) \int_0^1 dx \frac{ax^4 + 4x(1-x)}{ax^2 + bx + 1} \,, \tag{3.1}$$

where, Q_f is the charge and m_f is the mass of the external fermion, $m_{f'}$ is the mass of the internal fermion, $m_{Z'}$ is Z' boson mass, and $a = m_f^2/m_{Z'}^2$ and $b = (m_{f'}^2 - m_f^2)/m_{Z'}^2 - 1$. Note the contribution is non-zero only if both $B_{ff'}^L$ and $B_{ff'}^R$ are non-zero, at least one of them is complex, and their phases do not cancel. In the approximation of external quark mass being much less than the Z' mass, *i.e.* , $m_f \ll m_{Z'}$, the above equation can be simplified to be

$$d_f = -\frac{1}{8\pi^2} g_2^2 Q_f e \frac{m_{f'}}{m_{Z'}^2} \mathrm{Im}(B_{ff'}^{L}{}^* B_{ff'}^R) \frac{1 - c^2 + 2c \log c}{(1-c)^3} \,, \tag{3.2}$$

in which, $c = m_{f'}^2/m_{Z'}^2$. If the internal quark mass is again much less than $m_{Z'}$, *i.e.* , $m_{f'} \ll m_{Z'}$, then the equation can be further simplified into

$$\begin{aligned} d_f &= -\frac{1}{8\pi^2} g_2^2 Q_f e \frac{m_{f'}}{m_{Z'}^2} \mathrm{Im}({B^L_{ff'}}^* B^R_{ff'}) \\ &= -\frac{g_1^2}{8\pi^2} Q_f e \frac{m_{f'}}{m_Z^2}\, y\, \mathrm{Im}({B^L_{ff'}}^* B^R_{ff'})\,. \end{aligned} \tag{3.3}$$

where parameter y is defined as

$$y \equiv \left(\frac{g_2}{g_1}\right)^2 \frac{M_Z^2}{M_{Z'}^2}\,. \tag{3.4}$$

Now we can apply this result to electron and u and d quark EDMs. For electrons, both the diagrams with internal μ and internal τ contribute. By requiring both contributions to be less than electron EDM constraint $d_e < 1.4 \times 10^{-27} \mathrm{e}\cdot\mathrm{cm}$ [15], we get

$$y\, \mathrm{Im}({B^L_{e\mu}}^* B^R_{e\mu}) < 1 \times 10^{-6}\,, \tag{3.5}$$

$$y\, \mathrm{Im}({B^L_{e\tau}}^* B^R_{e\tau}) < 7 \times 10^{-8}\,. \tag{3.6}$$

The constraint on $B_{e\tau}$ is stronger simply because the contributions to the EDMs are proportional to the internal fermion masses.

The strongest bounds on $B^L_{e\mu}$ and $B^R_{e\mu}$ come from the non-observation of coherent μ-e conversion [6] by the Sindrum-II Collaboration [17], as the small mixing between Z and Z' can induce such conversion process,

$$w^2(|B^L_{e\mu}|^2 + |B^R_{e\mu}|^2) < 4 \times 10^{-14}\,, \tag{3.7}$$

where $w = g_2/g_1 \sin\theta\cos\theta(1 - m_Z^2/m_{Z'}^2)$ and θ is the Z-Z' mixing angle. In the most interesting case of a TeV-scale Z' with small mixing, $\theta \lesssim 10^{-3}$, y and w are of the same order, and $y \approx w \approx 10^{-3}$. This is the case we assume in comparing the constraints from different processes. It is difficult to directly compare the constraints in eq. (3.5) and eq. (3.7), since the former depends on the phase difference between $B^L_{e\mu}$ and $B^R_{e\mu}$ and the later on the absolute values. As $|B^{L,R}_{e\mu}|$ become as small as in eq. (3.7), the constraint in eq. (3.5) becomes unimportant. In this sense, we say the coherent μ-e conversion provides a stronger constraint on $B^{L,R}_{e\mu}$ than the electron EDM. The decay $\tau \to 3e$ provides the best constraint on flavor violating $Z'e\tau$ coupling

$$w^2(|B^L_{e\tau}|^2 + |B^R_{e\tau}|^2) < 2 \times 10^{-5}\,. \tag{3.8}$$

In this case, the constraint from electron EDM, eq. (3.6), is more stringent, although it depends on the phases.

We can also apply the same constraint on quark EDMs inferred from the neutron EDM. Barring possible cancellations, we require each diagram contributes less than the experimental limit $d_n < 3.0 \times 10^{-26}\mathrm{e}\cdot\mathrm{cm}$ [16], we get the following constraints

$$y\,\mathrm{Im}(B_{uc}^{L\,*}B_{uc}^{R}) < 3\times 10^{-6}\,, \tag{3.9}$$
$$y\,\mathrm{Im}(B_{ds}^{L\,*}B_{ds}^{R}) < 5\times 10^{-5}\,, \tag{3.10}$$
$$y\,\mathrm{Im}(B_{ut}^{L\,*}B_{ut}^{R}) < 2\times 10^{-8}\,, \tag{3.11}$$
$$y\,\mathrm{Im}(B_{db}^{L\,*}B_{db}^{R}) < 2\times 10^{-6}\,. \tag{3.12}$$

eq. (3.10) gives a weaker bound on $B_{ds}^{L,R}$ than those from $K_L \to \mu^+\mu^-$ [18] and $K_L \to \pi^0\mu^+\mu^-$ [19] decays

$$\begin{aligned} w^2|\mathrm{Re}B_{ds}^R - \mathrm{Re}B_{ds}^L|^2 &< 3\times 10^{-11}\,,\\ w^2|\mathrm{Im}B_{ds}^R + \mathrm{Im}B_{ds}^L|^2 &< 5\times 10^{-11}\,. \end{aligned} \tag{3.13}$$

At the same time, the constraint in eq. (3.12) are relevant, compared to bounds that come from B^0 decay into a $\mu^+\mu^-$ pair [20],

$$w^2|B_{db}^{L,R}|^2 < 10^{-5}\,. \tag{3.14}$$

The same diagram in figure 1, with the external fermion being μ, will contribute to muon $g-2$. The general expression for the contributions from the Z' diagram is given in ref. [21]. As the external and internal leptons masses are far smaller than the Z' mass, the dominant contribution becomes

$$a_\mu^{Z'} = -\frac{y}{4\pi^2}\frac{g_1^2}{m_Z^2}m_\mu m_\tau \mathrm{Re}(B_{\mu\tau}^{L\,*}B_{\mu\tau}^{R})\,. \tag{3.15}$$

If we demand this contribution to be less the the difference between the experimentally measured value and the Standard Model prediction [22], $\Delta a_\mu < 250\times 10^{-11}$, we get

$$y\,\mathrm{Re}(B_{\mu\tau}^{L\,*}B_{\mu\tau}^{R}) < 1\times 10^{-2}\,. \tag{3.16}$$

In the aforementioned small mixing and TeV-scale Z' case, this constraint on $B_{\mu\tau}$ is as strong as the one derived from $\tau\to 3\mu$ decay [6],

$$w^2(|B_{\mu\tau}^L|^2 + |B_{\mu\tau}^R|^2) < 10^{-5}\,. \tag{3.17}$$

The contribution from the diagram with electron in the loop are suppressed by the much lighter electron mass, thus it does not provide a useful constraint.

4. K-$\overline{K}$ mixing

The off-diagonal element M_{12} in the neutral K-$\overline{K}$ mixing mass matrix is related to the $|\Delta S| = 2$ effective Hamiltonian by

$$2m_K M_{12}^* = \langle \overline{K}^0|\mathcal{H}_{\mathrm{eff}}^{|\Delta S|=2}|K^0\rangle\,. \tag{4.1}$$

With the definition

$$\langle \overline{K}^0|[\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma^\mu(1-\gamma_5)d]|K^0\rangle \equiv \frac{8}{3}B_K f_K^2 m_K^2\,, \tag{4.2}$$

one obtains within the SM [23]

$$M_{12}^{\rm SM} = \frac{G_F^2}{16\pi^2} M_W^2 \left[(V_{cd}^* V_{cs})^2 \eta_1 S_0(x_c) + (V_{td}^* V_{ts})^2 \eta_2 S_0(x_t) + 2 (V_{cd}^* V_{cs})(V_{td}^* V_{ts}) \eta_3 S_0(x_c, x_t) \right] \times \frac{4}{3} B_K^{LL} f_K^2 m_K \ , \tag{4.3}$$

where the QCD factors $\eta_1 \simeq 1.38$, $\eta_2 \simeq 0.57$, and $\eta_3 \simeq 0.47$, and the Inami-Lim functions [25] $S_0(x)$ and $S_0(x, y)$ can be found, for example, in ref. [23]. The renormalization scale and scheme invariant bag parameter is

$$B_K^{LL} = \alpha_s^{(4)}(\mu)^{-2/9} \left[1 + 1.895 \frac{\alpha_s^{(4)}(\mu)}{4\pi} \right] B_K^{LL}(\mu) \,, \tag{4.4}$$

with the same factor for left-right mixing bag parameters B_{K1}^{LR} and B_{K2}^{LR}. We will take the following numerical values: $B_K^{LL}(2\ \text{GeV}) = 0.69 \pm 0.21$, $B_{K1}^{LR}(2\ \text{GeV}) = 1.03 \pm 0.06$, and $B_{K2}^{LR}(2\ \text{GeV}) = 0.73 \pm 0.10$ [26], $f_K = 159.8 \pm 1.5$ MeV, and $m_K = 497.648 \pm 0.022$ MeV [18]. The short-distance contribution to the mass difference between the two mass eigenstates of kaons, ΔM_K, is

$$\Delta M_K^{\rm SD} = 2 {\rm Re} M_{12} \ . \tag{4.5}$$

The CP violation is measured by the parameter

$$\epsilon_K = \frac{e^{i\pi/4}}{\sqrt{2} \Delta M_K} {\rm Im} M_{12} \ . \tag{4.6}$$

Their experimental values are given in [18] as $\Delta M_K = 0.5292 \times 10^{10}\ \text{s}^{-1}$ and $|\epsilon_K| = 2.284 \times 10^{-3}$.

The general set of $|\Delta S| = 2$ operators relevant for our discussions is:

$$\begin{aligned} O^{LL} &= [\bar{s}\gamma_\mu (1-\gamma_5) d][\bar{s}\gamma^\mu (1-\gamma_5) d] \ , \\ O_1^{LR} &= [\bar{s}\gamma_\mu (1-\gamma_5) d][\bar{s}\gamma^\mu (1+\gamma_5) d] \ , \\ O_2^{LR} &= [\bar{s}(1-\gamma_5) d][\bar{s}(1+\gamma_5) d] \ , \\ O^{RR} &= [\bar{s}\gamma_\mu (1+\gamma_5) d][\bar{s}\gamma^\mu (1+\gamma_5) d] \ . \end{aligned} \tag{4.7}$$

As seen previously, only the operator O^{LL} contributes to K-$\overline{K}$ mixing in the SM due to its chiral structure. The other three operators appear in the Z' models because the left- and right-handed couplings and operators mix through renormalization group (RG) evolution. The RG running of the Wilson coefficients $C(\mu)$ from the M_W scale down to the lattice scale μ_L, where we match with the lattice results of the associated hadronic matrix elements, can be schematically written as

$$C(\mu_L) = {\rm U}(\mu_L, M_W) C(M_W) \ . \tag{4.8}$$

Details of computing the evolution matrix ${\rm U}(\mu_L, M_W)$ are given in ref. [24]. Here we only provide the numerical values of the relevant evolution matrices:

$$U_{LL}^K = U_{RR}^K \simeq 0.788 \ , \tag{4.9}$$

$$U_{LR}^K = U_{RL}^K \simeq \begin{pmatrix} 0.906 & -0.087 \\ -1.531 & 3.203 \end{pmatrix} \ . \tag{4.10}$$

In determining these results, we have used only the central value of $\alpha_s(M_Z) = 0.118$ for a 5-quark effective theory and chose the lattice scale $\mu_L = 2$ GeV. The additional contribution from the Z' model with only the left-handed couplings is

$$M_{12}^{LL} = \frac{G_F}{\sqrt{2}} y U_{LL}^K \left(B_{ds}^L\right)^2 \frac{4}{3} B_K^{LL} f_K^2 m_K\,, \tag{4.11}$$

and the expression that also includes the right-handed coupling is

$$\begin{aligned} M_{12}^{LR} = & \frac{G_F}{\sqrt{2}} y f_K^2 m_K \left\{ \frac{4}{3} U_{LL}^K \left(B_{ds}^L\right)^2 B_K^{LL} + \frac{4}{3} U_{RR}^K \left(B_{ds}^R\right)^2 B_K^{RR} \right. \\ & \left. + \left(\frac{m_K}{m_s + m_d} \right)^2 B_{ds}^L B_{ds}^R \left[-\frac{4}{3} U_{LR}^K(1,1) B_{K1}^{LR} + 2 U_{LR}^K(2,1) B_{K2}^{LR} \right] \right\}. \end{aligned} \tag{4.12}$$

If we constrain the contribution to ΔM_K from M_{12}^{LL} to be less than the currently measure experimental value, we get the bound,

$$y |\, \mathrm{Re}\left(B_{ds}^L\right)^2 | < 2 \times 10^{-8}\,. \tag{4.13}$$

After including the RR and the LR mixing terms, the constraint becomes

$$y | 0.01\, \mathrm{Re}\left[\left(B_{ds}^L\right)^2 + \left(B_{ds}^R\right)^2 \right] - \mathrm{Re}\left(B_{ds}^L B_{ds}^R\right) | < 2 \times 10^{-10}\,. \tag{4.14}$$

Keeping the dominant term, we can simplify the equation into

$$y | \mathrm{Re}\left(B_{ds}^L B_{ds}^R\right) | < 2 \times 10^{-10}\,. \tag{4.15}$$

The theoretical uncertainty on ϵ_K within the Standard Model is mainly due to the uncertainty of the bag parameter B_K and it is estimated to be about 30% [26]. If we require that the contribution from Z' is less than the theoretical error associated with the SM prediction, we have

$$\begin{aligned} \frac{|\epsilon_K^{Z'}|}{|\epsilon_K^{exp}|} = & \frac{|\mathrm{Im} M_{12}^{Z'}|}{\sqrt{2} \Delta M_K^{\exp} \epsilon_K^{\exp}} \\ & \approx 1 \times 10^{12}\, y\, | 0.01\, \mathrm{Im}\left[\left(B_{ds}^L\right)^2 + \left(B_{ds}^R\right)^2 \right] - \mathrm{Im}\left(B_{ds}^L B_{ds}^R\right) | < 0.3\,. \end{aligned} \tag{4.16}$$

Assuming that only the LL coupling exists, the constraint becomes

$$y\, \mathrm{Im}\left(B_{ds}^L\right)^2 < 3 \times 10^{-11}\,. \tag{4.17}$$

When both left-handed and right-handed couplings contribute, we can ignore the LL and RR terms, and obtain

$$y\, \mathrm{Im}\left(B_{ds}^L B_{ds}^R\right) < 3 \times 10^{-13}\,. \tag{4.18}$$

In comparison with ref. [28], the stronger bound here is due to the chiral and renormalization enhancement in the left-right mixing terms.

5. B_d-$\overline{B}_d$ mixing

Similar to K-$\overline{K}$ mixing, chiral couplings of Z' with b and d quarks can induce B_d-$\overline{B}_d$ mixing. In the SM, the off-diagonal element in the B_d meson mass matrix is given by [23]

$$M_{12}^{\rm SM} = \frac{G_F^2}{16\pi^2} M_W^2 (V_{tb}^* V_{td})^2 \eta_B \frac{4}{3} B_B^{LL} f_B^2 m_B S_0(x_t)\,, \tag{5.1}$$

where the QCD factor $\eta_B \simeq 0.55$, $S_0(x_t) = 2.463$ [23], and $m_B = 5.2794 \pm 0.0005\,\mathrm{GeV}$ [18]. The renormalization scale invariant bag parameter is

$$B_B^{LL} = \alpha_s^{(5)}(\mu)^{-6/23} \left[1 + 1.627 \frac{\alpha_s^{(5)}(\mu)}{4\pi} \right] B_B^{LL}(\mu)\ , \tag{5.2}$$

with similar expressions for B_{B1}^{LR} and B_{B2}^{LR}, which are bag parameters for left-right mixing operators. The bag parameters in the $\overline{\rm MS}$ scheme are evaluated on the lattice with quenched approximation [29] and they are: $B_B^{LL}(4.6\ \mathrm{GeV}) = 0.87{\pm}0.06$, $B_{B1}^{LR}(4.6\ \mathrm{GeV}) = 1.72{\pm}0.12$, and $B_{B2}^{LR}(4.6\ \mathrm{GeV}) = 1.15 \pm 0.6$. The decay constant is $f_B = 173 \pm 23$ MeV.

It is a common practice to determine $\sin 2\beta$ from the time-dependent CP asymmetry of the $b \to c\bar{c}s$ processes because the decay amplitudes are dominated by tree-level processes and therefore least affected by new physics contributions [30]. Within the Standard Model, $\sin 2\beta$ is related to the CKM matrix elements

$$\beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right)\ . \tag{5.3}$$

Both ΔM_B and $\sin 2\beta$, determined from all charmonium modes, are measured at Belle [31] and BaBar [32] and the world average [33] are

$$\Delta M_B = 0.507 \pm 0.005\,\mathrm{ps}^{-1}\,, \tag{5.4}$$

$$\sin 2\beta = 0.687 \pm 0.032\ . \tag{5.5}$$

A set of $|\Delta B| = 2$ operators can be obtained by simply replacing $\bar{s}$ with $\bar{b}$ in eq. (4.7). Following ref. [24], we calculate the evolution matrices,

$$U_{LL}^B = U_{RR}^B \simeq 0.842\,, \tag{5.6}$$

$$U_{LR}^B = U_{RL}^B \simeq \begin{pmatrix} 0.921 & -0.041 \\ -0.882 & 2.081 \end{pmatrix}\ . \tag{5.7}$$

The contributions from Z' with purely left-handed couplings and with both left-handed and right-handed couplings to the off-diagonal M_{12}^B are similar to eq. (4.11) and eq. (4.12) with simple replacements of parameters

$$M_{12}^{LL} = \frac{G_F}{\sqrt{2}} y U_{LL}^B \left(B_{db}^L\right)^2 \frac{4}{3} B_B^{LL} f_B^2 m_B\,, \tag{5.8}$$

$$\begin{aligned} M_{12}^{LR} = \frac{G_F}{\sqrt{2}} y f_B^2 m_B \Bigg\{ & \frac{4}{3} U_{LL}^B \left(B_{db}^L\right)^2 B_B^{LL} + \frac{4}{3} U_{RR}^B \left(B_{db}^R\right)^2 B_B^{RR} \\ & + \left(\frac{m_B}{m_d + m_b}\right)^2 \left(B_{db}^L B_{db}^R\right) \left[-\frac{4}{3} U_{LR}^B(1,1) B_{B1}^{LR} + 2 U_{LR}^B(2,1) B_{B2}^{LR} \right] \Bigg\}\ . \end{aligned} \tag{5.9}$$

In the presence of Z' contributions, the weak phase thus measured should be an effective one, with

$$\beta_{\text{eff}} = -\frac{1}{2}\arg(M_{12}^{\text{SM}} + M_{12}^{Z'}) \ . \tag{5.10}$$

The measured ΔM_B and $\sin 2\beta_{\text{eff}}$ may both contain contributions from Z'. Therefore, assuming the existence of new physics, they cannot be used to determine the SM V_{td}. Without the accurate determination from B_d mixing and decay, information on the Wolfenstein parameters ρ and η [34] can only be derived from two sources. On the one hand, we can deduce constraints on $\sqrt{\rho^2+\eta^2}$ from $|V_{ub}| = (4.05\pm0.52)\times10^{-3}$, $|V_{cb}| = (41.4\pm2.1)\times10^{-3}$ and $|V_{cd}| = 0.224\pm0.014$ [35] and allow ρ and η values to vary within this constraint. $|V_{ub}|$ and $|V_{cb}|$ are determined from semileptonic decays of B mesons. $|V_{cd}|$ can be deduced from neutrino and antineutrino production of charm off valence d quarks. From $|V_{cd}|$, $|V_{cb}|$ and $|V_{ub}|$ we have

$$\left|\frac{V_{ub}}{V_{cd}V_{cb}}\right| = |\rho - i\eta| = 0.437 \pm 0.066\,. \tag{5.11}$$

On the other hand, the ratio $|V_{td}/V_{ts}|$ has recently been determined at BELLE [7] through $b\to d\gamma$ decays. Its value is found to be within the interval of $0.142 < |V_{td}/V_{ts}| < 0.259$ at a 95% confidence level. More importantly, diagrams involving Z' that contribute to the $b\to d\gamma$ or $b\to s\gamma$ process are not only loop suppressed but also mass suppressed. Therefore, the bound on $|V_{td}/V_{ts}|$ provides an additional constraint on the SM ρ and η parameter space. The constraint derived from $|V_{td}/V_{ts}|$, combined with $|V_{cd}|$, gives the ratio and its 1σ range

$$\left|\frac{V_{td}}{V_{cd}V_{ts}}\right| = |1-\rho - i\eta| = 0.888 \pm 0.163\,. \tag{5.12}$$

Note we have used the 95% confidence level bound to derive the 1σ error and turned asymmetric errors to symmetric ones assuming they are Gaussian.

In $\Delta M_d^{\text{exp}}/\Delta M_s^{\text{exp}}$, the ratio of the hadronic parameters $(f_{B_d}^2 B_{B_d})/(f_{B_s}^2 B_{B_s})$ is more accurately known than individual hadronic parameters. It may seem that the ratio would provide a better determination of the related Z' couplings. This is not so when Z' effects enter both ΔM_d and ΔM_s. While trying to constrain B_{db} from the ratio, we need to know B_{sb}. The hadronic uncertainties re-enter in the form of uncertainty on B_{sb} [12]. Hence, we use ΔM_{db} and $\sin 2\beta_{eff}$ to constrain B_{db}, together with the bounds on ρ and η discussed above. Note that, in the B_s system, the CP-violating parameter $\sin 2\phi_s$ can be combined with ΔM_{B_s} to determine new physics parameters [11, 13].

To be specific, we consider only the LL couplings in the following discussions. We now rewrite M_{12} in simple forms of ρ, η and B_{db},

$$\begin{aligned} M_{12}^{\text{SM}} &= \frac{G_F^2}{12\pi^2} M_W^2 \eta_B B_{B_d}^{LL} f_{B_d}^2 m_{B_d} S_0(x_t)(V_{tb}^* V_{td})^2 \\ &= C_1[1-(\rho+i\eta)]^2\,, \end{aligned} \tag{5.13}$$

$$\begin{aligned} M_{12}^{LL} &= \frac{G_F}{\sqrt{2}} y U_{LL}^B \left(B_{db}^L\right)^2 \frac{4}{3} B_{B_d}^{LL} f_{B_d}^2 m_{B_d} \\ &= C_2\, y \left(B_{db}^L\right)^2 \ , \end{aligned} \tag{5.14}$$

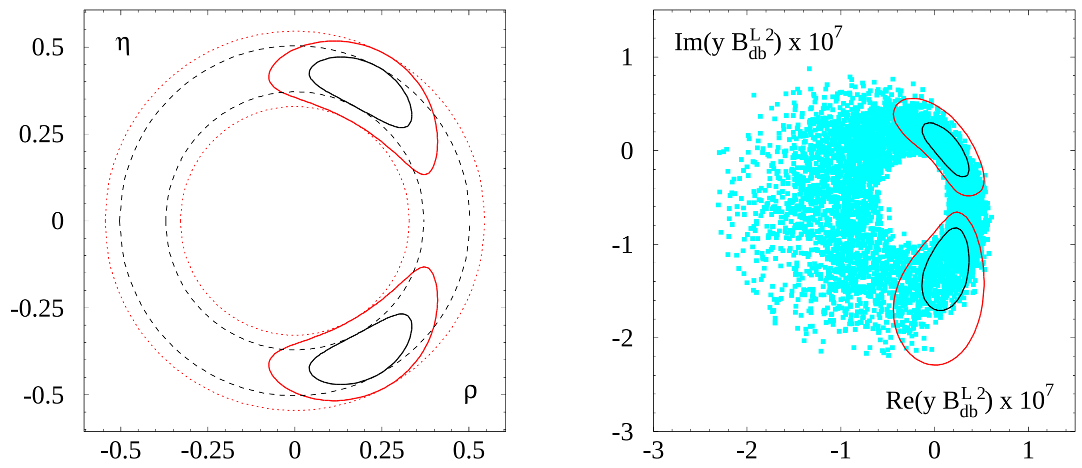

Figure 2: The allowed ρ and η values (left) when both SM and Z' contribute to B_d mixing, and the allowed ranges for $y\mathrm{Im}(B^L_{db})^2$ and $y\mathrm{Re}(B^L_{db})^2$ (right). For the dashed (1σ) and dotted (1.64σ) contours on the left and the scattered points (1σ) on the right, the constraint on $|V_{td}/V_{ts}|$ is not imposed. For the solid contours, black for 1σ and red for 1.64σ, the constraint on $|V_{td}/V_{ts}|$ is imposed.

with

$$C_1 \equiv \frac{G_F^2}{12\pi^2} M_W^2 \eta_B B^{LL}_{B_d} f^2_{B_d} m_{B_d} S_0(x_t) A^2 \lambda^6 = (1.823 \pm 0.52) \times 10^{-13}\ \mathrm{GeV}\,, \quad (5.15)$$

$$C_2 \equiv \frac{4}{3}\frac{G_F}{\sqrt{2}} U^B_{LL} B^{LL}_{B_d} f^2_{B_d} m_{B_d} = (1.947 \pm 0.53) \times 10^{-6}\ \mathrm{GeV}\,, \quad (5.16)$$

and the ratio $C_2/C_1 = (1.068 \pm 0.085) \times 10^7$. Here we take $A = 0.801 \pm 0.024$ and $\lambda = 0.2262 \pm 0.0020$ [35]. The observed ΔM_B and $\sin 2\beta$ render

$$2C_1 \left| [1-(\rho+i\eta)]^2 + \frac{C_2}{C_1}\, y\left(B^L_{db}\right)^2 \right| = (3.337 \pm 0.033) \times 10^{-13}\ \mathrm{GeV}\,, \quad (5.17)$$

$$-\arg\{[1-(\rho+i\eta)]^2 + \frac{C_2}{C_1}\, y\left(B^L_{db}\right)^2\} = 0.757^{+0.045}_{-0.043}\,. \quad (5.18)$$

We try to get limits on $y(B_{db})^2$ based on the four conditions in eqs. (5.11), (5.12), (5.17) and (5.18).

In figure 2, we show the allowed ranges in the ρ-η plane and in the plane of the Z' parameters $y\mathrm{Re}(B^L_{db})^2$ and $y\mathrm{Im}(B^L_{db})^2$. We show the results before imposing the $|V_{td}/V_{ts}|$ constraint, *i.e.* , eq. (5.12), with the dashed (1σ) and dotted (1.64σ) contours in the left plot, and with the scattered points in the right plot. Because of the additional Z' contributions, ρ and η are allowed to take all possible values allowed by the $|V_{ub}|$, $|V_{cd}|$ and $|V_{cb}|$ measurements, and the corresponding allowed range for Z' parameters are approximately $-2 \times 10^{-7} < y\mathrm{Re}(B^L_{db})^2 < 1 \times 10^{-7}$ and $-2 \times 10^{-7} < y\mathrm{Im}(B^L_{db})^2 < 1 \times 10^{-7}$. However, after imposing the $|V_{td}/V_{ts}|$ constraint, the allowed region of ρ and η as well as those of $y\mathrm{Re}(B^L_{db})^2$ and $y\mathrm{Im}(B^L_{db})^2$ improve significantly, as shown by the solid black (1σ) and red

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(1.64σ) contours in both plots of figure 2. Just from the four conditions listed above, the $\eta < 0$ region is allowed, leaving a two-fold ambiguity on the allowed regions. Under the assumption that Z' is not the dominant contribution in ϵ_K, we can use the ϵ_K measurement to exclude the $\eta < 0$ region. From another point of view, for the lower regions in both plots, the large Z' contributions have to be canceled by the SM contributions to reproduce ΔM_B and $\sin 2\beta$ measurements. The lower regions are thus less natural, and we limit ourself to the upper regions. Hence, when only left-handed couplings are present, the bounds can be estimated from the right plot of figure 2 to be

$$y|\mathrm{Re}(B^L_{db})^2| < 5 \times 10^{-8}\,, \tag{5.19}$$

$$y|\mathrm{Im}(B^L_{sd})^2| < 5 \times 10^{-8}\;. \tag{5.20}$$

When both left-handed and right handed couplings are included, the constraints are

$$y|\mathrm{Re}[(B^L_{db})^2 + (B^R_{db})^2] - 3.8\mathrm{Re}(B^L_{db}B^R_{db})| < 5 \times 10^{-8}\,, \tag{5.21}$$

$$y|\mathrm{Im}[(B^L_{db})^2 + (B^R_{db})^2] - 3.8\mathrm{Im}(B^L_{db}B^R_{db})| < 5 \times 10^{-8}\,, \tag{5.22}$$

which are less illuminating because of the possible cancellation among different terms.

6. Conclusions

Flavor-changing and CP-violating processes are natural consequences of family-nonuniversal Z' models, and they can manifest in observables such as EDMs, muon $g-2$ and meson mixings. We have studied constraints on Z' couplings from electron and neutron EDMs, muon $g-2$, K and B meson mixing and CP violation.

We presented the general expression for the fermion EDM generated by a one-loop diagram induced by the Z' boson. In the approximation that both internal and external fermion masses are much smaller than the Z' mass, the EDM is a simple quantity proportional to Z' couplings and the internal fermion mass. We obtained the constraints on the chiral couplings to Z' by requiring each individual contribution to be within the experimental limits of electron and neutron EDMs. Derived from the electron EDM, the constraint on $B^{L,R}_{e\mu}$ is weaker than that from the μ-e conversion, while the constraint on $B^{L,R}_{e\tau}$ is stronger than that from the $\tau \to 3e$ decay. From the neutron EDM, bounds on $B^{L,R}_{ds}$ are not as strong as those imposed by the $K_L \to \mu^+\mu^-$ and $K_L \to \pi^0\mu^+\mu^-$ decays. However, bounds on $B^{L,R}_{db}$ are stronger than bounds from the B^0 to $\mu^+\mu^-$ decay. Because the EDMs are proportional to the internal fermion masses, they provide better constraints on couplings involving heavier leptons and quarks. Requiring the Z' contribution to muon $g-2$ to be less than the discrepancy between theoretical and experimental values, we obtained comparable limits on $B^{L,R}_{\mu\tau}$ to that from the $\tau \to 3\mu$ decay.

We calculated the K-$\bar{K}$ mixing mass difference and the CP-violating parameter ϵ_K. Due to the enhancement in the left-right mixing terms, their coefficients are two orders of magnitude bigger than those of purely left-handed and right-handed terms. Therefore, the constraint on the product $B^L_{ds}B^R_{ds}$ is much stronger than those on $(B^L_{ds})^2$ and $(B^R_{ds})^2$. The

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mass difference provides a limit on the real part of $B^L_{ds}B^R_{ds}$, while the ϵ_K provides a limit on its imaginary part.

We also evaluated the B_d-$\bar{B}_d$ mixing in the context of the flavor-changing Z' couplings. Because the measured mass difference and CP asymmetry may partially involve new physics at present, we can no longer use the V_{td} determined from the B_d-$\bar{B}_d$ system assuming only the SM physics. Instead, V_{td} is relaxed to all possible values allowed by the unitarity triangle, with $|V_{ub}|$, $|V_{cb}|$ and $|V_{cd}|$ fixed by the semileptonic B decays and the neutrino and anti-neutrino production of charm. Furthermore, because Z' contributions to $b \to s\gamma$ and $b \to d\gamma$ decays are both loop and mass suppressed, these processes can be used to constrain the SM $|V_{td}/V_{ts}|$. We used such limits to improve the analysis on B_d mixing. We found that there was only a small window for the Z' physics in the B_d system when one took into account all the constraints on ΔM_B, $\sin 2\beta$, $|V_{ub}|$, and $|V_{td}/V_{ts}|$ given by the different experiments.

Acknowledgments

We thank Paul Langacker for helpful discussions. C.-W.C. is grateful for the hospitality of the National Center for Theoretical Sciences at Hsinchu, Taiwan. This research was supported in part by the National Science Council of Taiwan, R.O.C. under Grant No. NSC 94-2112-M-008-023- and in part by the U.S. Department of Energy under Grants No DE-FG02-96ER40969.

References

[1] S. Chaudhuri, S.W. Chung, G. Hockney and J.D. Lykken, *String consistency for unified model building*, *Nucl. Phys.* **B 456** (1995) 89 [hep-ph/9501361];
G. Cleaver et al., *Physics implications of flat directions in free fermionic superstring models, I. Mass spectrum and couplings*, *Phys. Rev.* **D 59** (1999) 055005 [hep-ph/9807479];
M. Cvetič, G. Shiu and A.M. Uranga, *Three-family supersymmetric standard-like models from intersecting brane worlds*, *Phys. Rev. Lett.* **87** (2001) 201801 [hep-th/0107143];
M. Cvetič, P. Langacker and G. Shiu, *Phenomenology of a three-family standard-like string model*, *Phys. Rev.* **D 66** (2002) 066004 [hep-ph/0205252].

[2] See, for example, M. Masip and A. Pomarol, *Effects of SM Kaluza-Klein excitations on electroweak observables*, *Phys. Rev.* **D 60** (1999) 096005 [hep-ph/9902467].

[3] CDF collaboration, F. Abe et al., *Search for new gauge bosons decaying into dileptons in $\bar{p}p$ collisions at $\sqrt{s} = 1.8$ TeV*, *Phys. Rev. Lett.* **79** (1997) 2192;
For more recent result see, http://www-cdf.fnal.gov/~harper/diEleAna.html.

[4] J. Erler and P. Langacker, *Constraints on extended neutral gauge structures*, *Phys. Lett.* **B 456** (1999) 68 [hep-ph/9903476];
G.-C. Cho, K. Hagiwara and Y. Umeda, *Z' bosons in supersymmetric E_6 models confront electroweak data*, *Nucl. Phys.* **B 531** (1998) 65 [hep-ph/9805448].

[5] V.D. Barger et al., *Discovery limits of new gauge bosons of* $\mathrm{SP}_L(6) \times \mathrm{U}(1)$, *Int. J. Mod. Phys.* **A 2** (1987) 1327.

[6] P. Langacker and M. Plumacher, *Flavor changing effects in theories with a heavy Z' boson with family non-universal couplings*, *Phys. Rev.* **D 62** (2000) 013006 [hep-ph/0001204].

[7] BELLE collaboration, D. Mohapatra et al., Belle Preprint 2006-5.

[8] D0 collaboration, V.M. Abazov et al., *First direct two-sided bound on the B_s^0 oscillation frequency*, *Phys. Rev. Lett.* **97** (2006) 021802 [hep-ex/0603029].

[9] CDF Run II collaboration, G. Gomez-Ceballos *B_s mixing at the Tevatron*, FERMILAB-CONF-06-076-E, presented at 20th Rencontres de Physique de La Vallee d'Aoste: *Results and Perspective in Particle Physics, La Thuile, Aosta Valley, Italy, 5-12 Mar 2006.*

[10] M. Blanke, A.J. Buras, D. Guadagnoli and C. Tarantino, *Minimal flavour violation waiting for precise measurements of ΔM_s, $|V_{ub}|$, γ and B_s, $D^0 \to \mu^+\mu^-$*, hep-ph/0604057;
Z. Ligeti, M. Papucci and G. Perez, *Implications of the measurement of the B_s^0 - $\bar{B}_s^0$ mass difference*, hep-ph/0604112;
A. Datta, *B_s mixing and new physics in hadronic $B \to s\bar{q}q$ transitions*, *Phys. Rev.* **D 74** (2006) 014022 [hep-ph/0605039].

[11] V. Barger, C.-W. Chiang, J. Jiang and P. Langacker, *$B_s - \bar{B}_s$ mixing in Z' models with flavor-changing neutral currents*, *Phys. Lett.* **B 596** (2004) 229 [hep-ph/0405108].

[12] K. Cheung, C.-W. Chiang, N.G. Deshpande and J. Jiang, *Constraints on flavor-changing Z' models by B_s mixing, Z' production and $B_s \to \mu^+\mu^-$*, hep-ph/0604223.

[13] P. Ball and R. Fleischer, *Probing new physics through B mixing: status, benchmarks and prospects*, hep-ph/0604249.

[14] X.-G. He and G. Valencia, *$\bar{B}_s - B_s$ mixing constraints on FCNC and a non-universal Z'*, *Phys. Rev.* **D 74** (2006) 013011 [hep-ph/0605202].

[15] B.C. Regan, E.D. Commins, C.J. Schmidt and D. DeMille, *New limit on the electron electric dipole moment*, *Phys. Rev. Lett.* **88** (2002) 071805.

[16] C.A. Baker et al., *An improved experimental limit on the electric dipole moment of the neutron*, hep-ex/0602020.

[17] P. Wintz, *Results of the SINDRUM-II experiment*, *Prepared for 29th International Conference on High-Energy Physics (ICHEP 98), Vancouver, Canada, 23-29 Jul 1998.*

[18] Particle Data Group collaboration, S. Eidelman et al., *Review of particle physics*, *Phys. Lett.* **B 592** (2004) 1.

[19] KTeV/E832-E799 collaboration, J. Whitmore, *New rare decay results from KTeV*, FERMILAB-CONF-99-266-E.

[20] R.V. Kooten, talk presented at FPCP 2006, Vancouver, Canada, April 9-12, 2006.

[21] K.R. Lynch, *Extended electroweak interactions and the muon $g_\mu - 2$*, *Phys. Rev.* **D 65** (2002) 053006 [hep-ph/0108080].

[22] A. Czarnecki, *Muon $g - 2$: a theoretical review*, *Nucl. Phys.* **144** *(Proc. Suppl.)* (2005) 201.

[23] G. Buchalla, A.J. Buras and M.E. Lautenbacher, *Weak decays beyond leading logarithms*, *Rev. Mod. Phys.* **68** (1996) 1125 [hep-ph/9512380].

[24] M. Ciuchini et al., *Next-to-leading order QCD corrections to $\Delta F = 2$ effective hamiltonians*, *Nucl. Phys.* **B 523** (1998) 501 [hep-ph/9711402];
A.J. Buras, S. Jager and J. Urban, *Master formulae for $\Delta F = 2$ NLO-QCD factors in the standard model and beyond*, *Nucl. Phys.* **B 605** (2001) 600 [hep-ph/0102316].

[25] T. Inami and C.S. Lim, *Effects of superheavy quarks and leptons in low-energy weak processes $K_L \to \mu\bar{\mu}, K^+ \to \pi^+$ neutrino anti-neutrino and $K_0 \leftrightarrow \bar{K}_0$*, *Prog. Theor. Phys.* **65** (1981) 297.

[26] C.R. Allton et al., *B-parameters for $\Delta S = 2$ supersymmetric operators*, *Phys. Lett.* **B 453** (1999) 30 [hep-lat/9806016].

[27] R. Gupta, T. Bhattacharya and S.R. Sharpe, *Matrix elements of 4-fermion operators with quenched Wilson fermions*, *Phys. Rev.* **D 55** (1997) 4036 [hep-lat/9611023].

[28] X.-G. He and G. Valencia, *$K^+ \to \pi^+\nu\bar{\nu}$ and FCNC from non-universal Z' bosons*, *Phys. Rev.* **D 70** (2004) 053003 [hep-ph/0404229].

[29] D. Becirevic, V. Gimenez, G. Martinelli, M. Papinutto and J. Reyes, *B-parameters of the complete set of matrix elements of $\Delta B = 2$ operators from the lattice*, *JHEP* **04** (2002) 025 [hep-lat/0110091].

[30] See, however, W.-S. Hou, M. Nagashima and A. Soddu, *Direct CP-violation in $B^+ \to J_\psi K^+$ decay as probe for new physics*, hep-ph/0605080.

[31] BELLE collaboration, K. Abe et al., *Improved measurement of CP-violation parameters $\sin(2\phi_1)$ and $|\lambda|$, B meson lifetimes and $B^0 - \bar{B}^0$ mixing parameter δm_d*, *Phys. Rev.* **D 71** (2005) 072003 [hep-ex/0408111].

[32] BABAR collaboration, B. Aubert et al., *Improved measurement of CP asymmetries in $B^0 \to (c\bar{c})K^{0(*)}$ decays*, *Phys. Rev. Lett.* **94** (2005) 161803 [hep-ex/0408127].

[33] Heavy Flavor Averaging Group (HFAG) collaboration, *Averages of B-hadron properties at the end of 2005*, hep-ex/0603003.

[34] L. Wolfenstein, *Parametrization of the Kobayashi-Maskawa matrix*, *Phys. Rev. Lett.* **51** (1983) 1945.

[35] CKMfitter Group collaboration, J. Charles et al., *CP-violation and the CKM matrix: assessing the impact of the asymmetric B factories*, *Eur. Phys. J.* **C 41** (2005) 1 [hep-ph/0406184];
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